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ABSTRAKTY

High-velocity ballistics

Stanislav Barton

In the presented article, the fundamental equations of motion of a body moving at high speed in the atmosphere are first derived. Algebraic and numerical procedures for their solution are presented for these equations, and the whole problem is gradually generalised - from free fall in a resisting environment to the entry of a body at the second cosmic speed into the earth's atmosphere. In the end, the essential conditions necessary for the survival of the spaceship crew, which enters the Earth's atmosphere at the second cosmic speed after returning from the lunar mission, are derived.

Tvarová optimalizace lopatky vodní turbíny založená na adjungované metodě a isogeometrické analýze

Marek Brandner

V příspěvku bude prezentována spojitá verze adjungované metody pro optimalizaci tvaru lopatky vodní turbíny. Nestlačitelné vazké proudění je modelováno Navierovými-Stokesovými rovnicemi. Cílová funkce je navržena na základě požadavků zákazníka. Numerický model je založen na isogeometrické analýze. Podrobně budou rozebrány přístup založený na formulaci adjungované úlohy pomocí Ceovy formální techniky odvození a algoritmus optimalizační úlohy.

Tree-based QR factorization methods for tall-and-skinny matrices with task-based multicore implementation

Vít Břichňáč, **Jakub Šístek**

The need for computing the QR factorization of dense matrices with substantially more rows than columns (so-called tall-and-skinny matrices) arises in a number of numerical algorithms, for example, when solving overdetermined systems of linear equations by the least-squares method. We study several algorithms for QR factorization based on hierarchical Householder reflectors organized into elimination trees, which are particularly suited for tall-and-skinny matrices and allow parallelization. We examine the effect of various parameter values on the performance of the tree-based algorithms. The work is accompanied with a custom implementation that utilizes a task-based runtime system (OpenMP or StarPU). We compare the newly developed implementation with the existing OpenMP implementation from the PLASMA library. In the first part of the presentation, we describe basic concepts regarding the tree-based blocked algorithms for QR factorization. In the second part, we compare different parameter settings. We find that in order to achieve a good performance for tall-and-skinny matrices of different widths, the algorithm needs to adjust the parameter values to the particular number of processor cores. We also find that the optimal parameter values vary for different processor families and architectures (AMD, Intel and Arm). This is joint work with Jakub Šístek.

Shape optimization of Gao beam with respect to stiffness and stability

Jana Burkotová, **Jitka Machalová**

In this contribution, shape optimization problem of elastic beam described by nonlinear Gao beam model is considered. The goal of the optimization is to provide shape parameters leading to maximal stiffness and stability of the beam. The stiffness is defined by compliance cost functional and the stability is represented by minimal buckling load that causes non-convexity of the problem. Existence of solutions and convergence properties of the discretized problem are analysed and numerical realization is discussed.

Domain decomposition based preconditioners for discontinuous Galerkin methods

Vít Dolejší

The discontinuous Galerkin method (DGM) is an efficient numerical method for the solution of PDEs. In contrary to conforming methods, DGM employs a discontinuous piecewise polynomial approximation. The arising algebraic system are typically larger in comparison to conforming discretizations but the non-conformity of the DGM can be successfully exploited in the development of efficient algebraic solvers leading to situations which have non analogous in the conforming case. Particularly, the construction of multi-level domain decomposition (DD) preconditioners suitable for multiprocessor computers is direct and very efficient.
We introduce the main ideas of two-level DD preconditioners for DGM, namely in a comparison to the conforming case. We present some theoretical results related to the condition number of the preconditioned linear algebraic systems and mention its limits. Further, we discuss several implementation aspects of the DD method, particularly the construction of the coarse solvers. Finally, we demonstrate the computational performance of this approach by several numerical examples including nonlinear problems, time-dependent problems and computations with the anisotropic hp-mesh adaptation.

Galerkin-type solution of non-stationary aeroelastic stochastic problems

Cyril Fischer

The assessment of vibration characteristics in slender engineering structures, influenced by both deterministic harmonic and stochastic excitation, poses a challenging problem. Due to its complexity, transverse vibration of the structure (relative to the wind direction) is typically modeled using the single-degree-of-freedom van der Pol-type equation. Determining the response probability density function comprises solving the Fokker-Planck equation, a task that generally necessitates the use of approximate numerical methods. Some of these methods rely on Galerkin-type approximation employing orthogonal polynomial or exponential-polynomial basis functions. This contribution reviews available techniques and proposes new modifications while highlighting unresolved questions in the field.

Improving performance of augmented Lagrangians

Barbora Halfarová, David Horák, Zdeněk Dostál, Jakub Kružík

SMALSE-rho and SMALSE-M algorithms are efficient tools for solving the quadratic programming problems with equality constraint and simple bound. These algorithms consist of an outer loop for update of parameters rho or M, approximation of the Lagrange multipliers for equality constraint and MPRGP algorithm used as an inner solver for problems with penalized equality constraint and simple bound. Parameter M is fixed for SMALSE-rho while penalty rho increases depending on the augmented Lagrangian growth. In SMALSE-M depending on the augmented Lagrangian growth, M decreases while rho is fixed. The larger penalty rho accelerates an outer loop, while larger parameter M accelerates an inner one. The new theoretically supported SMALSE-rho, M variant increases, both M and rho parameters and reduces numbers of outer and inner iterations. The performance of SMALSE versions can be further essentially improved by enhancing the information on the free set of current iterates into the reorthogonalization of equality constraints. The poster deals with the efficient parallel implementation in PERMON requiring no transformation of the Lagrange multipliers and no assembling and no factorizing the whole product of matrices with equality constraints reflecting changed free set, but just their small submatrix dealing with rows affected by simple bound.

Domain decomposition method for discontinuous Galerkin method solving symmetric linear elliptic problem

Tomáš Hammerbauer

We solve the linear Laplace equation by the Discontinuous Galerkin method and the arising linear algebraic system are solved iteratively using a Domain decomposition method based preconditioners. In particular, we employ two-level additive Schwarz technique. The coarse mesh is obtained by the agglomeration of the elements of the fine mesh, which is another advantage of DGM. We present some theoretical results concerning the condition number of the preconditioned system related to the number of subdomains. The theoretical results are accompanied by numerical experiments related to the various discretization parameters and polynomial approximation degrees.

Efficient solution of incompressible flow problems using isogeometric analysis

Hana Honnerová

Isogeometric analysis (IgA) is a computational approach to solving problems modeled by partial differential equations, that has much in common with the popular finite element method. It is based on the idea of integrating methods for analysis into conventional NURBS-based geometric design tools. Therefore, following the isoparametric concept, we work with NURBS functions as discretization bases in IgA. This allows an exact representation of the domain boundary and introduces higher smoothness of the approximate solutions at the element boundaries.
Practical use of IgA is still not very common, especially in the field of computational fluid dynamics. In this talk, we will discuss its application to incompressible flow problems and some of the challenges we encountered during development of an efficient isogeometric incompressible flow solver. The addressed topics include the matrix assembly, solution of the arising linear systems and parallel implementation for use on clusters.

Nestochastická kvantifikace nejistoty v odezvě multi-modelu

Jan Chleboun

Přestože žijí ve stínu stochastických metod, i nestochastické přístupy mají své uplatnění při posuzování charakteru a rozsahu nejistoty v odezvě modelu závislého na nejistých vstupních datech. Zaměříme se na použití fuzzy množin a Dempsterovy-Shaferovy teorie. U fuzzy množin se stupeň příslušnosti prvku k množině vyjadřuje funkcí příslušnosti, což je zobecnění standardní charakteristické funkce množiny. Při vstupních fuzzy datech se i odezva modelu stává fuzzy množinou, cílem pak je odvodit (obvykle ovšem numericky aproximovat) její funkci příslušnosti. Dempsterova-Shaferova teorie používá jistou základní sadu předem stanovených množin s pevně danými vahami k vyhodnocení úrovně vztahu zvolené testované množiny k množinám základní sady. V přednášce bude mj. ukázáno, jak oba přístupy zkombinovat a aplikovat na situaci, kdy pro jeden jev je k dispozici více matematických modelů (multi-model) a zajímá nás soulad jejich odezvy na fuzzy vstupy. Situace s více modely není v technické praxi neobvyklá, například ve světě se celkové používají desítky modelů dlouhodobého chování betonu nebo připomeňme různé modely nosníků, např. Eulerův-Bernoulliho, Tymošenkův (s opravnými smykovými činiteli navrženými různými autory), Gaův aj.

Solution of the 2D/3D Signorini problem for the boundary modelled by the Bezier curve or Bezier surface

Lukáš Kapera

The task of solving shape optimization problems is to find the optimal shape of the boundary of a 2D or 3D body so that the given criterion described by the objective function is optimal. This boundary is modelled, for example, by a Bezier curve or Bezier surface defined by so-called control points. For a given Bezier curve or surface, a new mesh must always be generated for a given body, and the elasticity problem must be solved.

The contribution will introduce the design of a 2D/3D body boundary using a Bezier curve or Bezier surface and the subsequent solution of the Signorini problem with such a modelled boundary.

Numerical study of the optimal viscosity for the simulation of Newtonian fluid flow

Radka Keslerová

This paper deals with the problem of choosing an optimal viscosity value for a Newtonian fluid that can be used to approximate a non-Newtonian fluid. By using this approximation in the numerical simulation of non-Newtonian fluid flow, it is possible to reduce the computational time while maintaining reasonable accuracy of the numerical solution results.

The Euclidean norm of the velocity field difference is used to evaluate and compare the results of the Newtonian and non-Newtonian fluid simulations. For this study, the Carreau and the Power law viscosity models were chosen as reference samples. The viscosity for the Newtonian fluid is investigated within the range of the limiting viscosity values for the Carreau model.

The numerical simulation is tested on a channel geometry with idealised stenosis and with asymetric 33% and 66% stenosis with different refinement of the computational mesh. The calculations are performed using OpenFOAM.

Metoda vnitřního bodu pro kontaktní úlohy a úlohy proudění se skluzovou podmínkou

Radek Kučera

V přednášce se budeme zabývat metodou vnitřního bodu pro minimalizaci kvadratické funkce s jednoduchými a separovatelnými sférickými omezeními a s rovnostním omezením. Minimalizační úlohy tohoto typu vznikají při konečněprvkové aproximaci kontaktních úloh lineární pružnosti se třením nebo u úloh proudění se skluzovou okrajovou podmínkou. Bude ukázána varianta této metody spočívající ve sledování cesty, pro niž bude představena konvergenční analýza a implementace umožňující řešit rozsáhlé praktické úlohy. Pozornost bude věnována zejména řešení vnitřních blokových soustav lineárních rovnic, jejichž matice konvergují ke špatně podmíněné matic nebo dokonce k matici singulární. Tato nepříjemná vlastnost je důsledkem konstrukce algoritmu (nikoliv fyzikální úlohy, která stojí v pozadí) a lze ji odstranit vhodným předpodmíněním. Bude ukázáno, že předpodmíněné matice mají stejně omezené spektrum. Efektivitu metody budeme demonstrovat na výše zmíněných kontaktních úlohách nebo na úlohách se skluzem a porovnáme ji s dalšími možnostmi řešení těchto úloh.

Convergence of finite elements on extremely deformed meshes: optimal results and why we should care

Vašek Kučera

After more than 50 years of research, we are still unable to formulate a necessary and sufficient condition on triangular meshes for piecewise linear elements to converge, even for the simplest Poisson problem. The standard textbook minimum and maximum angle conditions are sufficient, but certainly not necessary. Here we present recent optimal analyses of special cases, especially the 'band of caps'. Furthermore, we introduce a simple modification that enables the finite element method to converge even on meshes violating these conditions. These seemingly academic considerations are now coming into the spotlight with the development of the X-MESH method of Remacle and Moës, which views extremely deformed meshes as an advantage, not a problem.

Finding a Hamiltonian cycle using the Chebyshev polynomials

Jan Lamač

The problem of finding a Hamiltonian cycle in a general undirected graph is one of the basic optimization tasks and has wide application not only in logistics, but also in some modern fields, such as computer graphics or microchip construction. However, it belongs to the so-called NP-complete problems and finding an algorithm that could solve NP-complete problems in polynomial time is one of the seven Millennium Prize Problems. In graph theory, there exists a number of sufficient conditions guaranteeing that a given graph is Hamiltonian (i.e. contains a Hamiltonian cycle). These conditions are most often based on some properties of the graph, such as the sum of degrees of non-adjacent vertices or the minimum degree of the graph. In this contribution we apply a different (numerical) approach: The characteristic polynomial of the adjacency (or Laplacian) matrix of an undirected graph formed by a single Hamiltonian cycle is related to some Chebyshev polynomial of the first kind. We use the properties of Chebyshev polynomials and present the algorithm consisting in finding a Hamiltonian cycle by minimization of an appropriately chosen functional.

Comparison of iterative solvers for poroelasticity

Tomáš Luber

Poroelastic models describe a materials undergoing elastic deformation while being permeated by a fluid flow. In this contribution, we consider a classical Biot's model that couples linear elasticity with saturated flow, as described by Darcy's law. The model finds many practical applications, from which we focus on geomechanics. The aim of this contribution is to compare two strategies for decoupling the solution of the model. More precisely, we will use implicit Euler method for time discretization and finite elements to discretize in space. The resulting matrix system will be solved by preconditioned Krylov space methods and by splitting methods, both effectively decouple the model and shift the main work to the solution of modified single field systems. These strategies will be compared on numerical experiments with settings motivated by geomechanical applications.

On condition number of the Steklov-Poincaré operator discretized by FEM and BEM

Dalibor Lukáš, P. Vodstrčil, Z. Dostál, M. Sadowská, D. Horák, J. Kružík, J. Bouchala, and O. Vlach

The finite element (FEM) and boundary element method (BEM) are two fundamental methods for the discretization of elliptic boundary value problems. Here we are interested in their application in the context of un-preconditioned FETI (finite element tearing and interconnecting) and BETI (boundary element) domain decomposition method for solving huge linear systems arising from the discretization of variational inequalities for elliptic partial differential equations. The reason for no preconditioning is that the duality transforms the general inequality constraints to bound constraints that can be solved very efficiently by the specialized algorithms, but the standard preconditioners transform the variables and do not preserve the bound constraints.
The local problems in FETI and BETI are defined by the Schur complements S_i^{FEM} and S_i^{BEM}. Since they both approximate the Steklov-Poincaré operator, it is natural to assume that that they are very similar to each other. Qualitatively, it is true. The condition numbers of both matrices are proportional to H/h, where H and h denote the diameter of the subdomain and the discretization parameter, respectively. However, closer inspection of the conditioning of S_i^{BEM} is more favorable than that of S_i^{FEM}. In this talk we shall prove that for a 2D model scalar problem the regular condition number κS_i^{BEM} is less than half of that of κS_i^{FEM} for $h \to 0$, which typically leads to reduction of conjugate gradient iterations by 30Similar behaviour is numerically observed for various domain shapes including an L-shape domain.

Metody nehladkých rovnic pro řešení obecných úloh nelineárního programování

Ladislav Lukšan

V příspěvku popíšeme metodu spádových směrů používající cenovou funkci, která je výpočetně velmi účinná, ale není obecně globálně konvergentní. Dále vyšetříme složitější metody nehladkých rovnic, které používají dvoukriteriální filtry a které jsou globálně a za vhodných předpokladů i lokálně a superlineárně konvergentní.

BEM preconditioner suitable for GPU implementation

Zbyšek Machaczek

In this talk, we propose a method, designed for GPU implementation, of preconditioning the single-layer potential operator, V, for the Laplace equation. The general idea is to use a suitable composition of the hypersingular operator, D, and the corresponding duality pairing, M, as our preconditioner. It is known that the analytical requirements of this approach are satisfied if we use piecewise constant functions for the approximation of V and piecewise linear functions on a mesh dual to that of V for the approximation of D. However, such approach is highly computationally inefficient. Our method avoids dealing with dual meshes and instead utilizes change of basis of a hp-method approximation of D.

Buckling of the Gao beam

Jitka Machalová

Our study is devoted to the original Gao static beam model, focusing on buckling phenomena under combined axial and transverse loads. Buckling analysis determines critical loads at which structures become unstable. While the Euler-Bernoulli model offers initial insights using eigenvalues, it is limited to small deformations. However, the nonlinear Gao beam model is used for moderately large deflections and post-buckling analysis. We will present the behavior of the Gao beam in the non-convex case and of bifurcation in static equilibrium. Through our study, we aim to illustrate the determination of critical loads and the complexity of post-buckling shapes with bifurcation diagrams.

Použití Tichonovy regularizace pro určení původního napětí pro úložiště radioaktivního odpadu

Josef Malík, Alexej Kolcun

Znalost tenzoru původního napětí hraje podstatnou roli ve všech úlohách geomechaniky. Velmi důležitou roli hraje při projektování úložiště radioaktivního odpadu. Stabilita ukládacích chodeb je sině závislá na orientaci těchtu chodeb vůči tenzoru původního napětí. Rovněž tvar profilu chodeb vůči tenzoru původního napětí ovlivňuje stabilitu úložiště. Určení tenzoru původního napětí je nedílnou součástí všech zahraničních projektů. Ve svém příspěvku jsme navrhli novou metodu určení tenzoru pomocí analýzy deformací na stěnách ukládací chodby při procesu hloubení. Metoda je založena na použití Tichonovy regulatizace a bude demonstrována na numerickém experimentu.

Numerical approximation of aeroacoustics induced by flow over a square cylinder

Tomáš Marhan

This study outlines a possible approach to aeroacoustics. An unsteady simulation of a 3D flow field in vicinity of a square cylinder is performed within the OpenFOAM framework using $k - \omega$ SST turbulence model. Afterwards, implementation of Lighthill's acoustic analogy is discussed using FEM. The study concludes by presenting results both in the fluid and acoustic domains.

Jeden (ne úplně tradiční) pohled na Lambertovu W funkci

Luděk Nechvátal

Lambertova W funkce je speciální (komplexní a mnohoznačnou) funkcí. Ačkoli se její počátky vážou k ryze teoretické úloze, v dnešní době je známo, že má poměrně široký aplikační potenciál. Zajímavostí je, že ačkoli se jedná o komplexní funkci, v jistém smyslu s ní lze manipulovat v reálném oboru. Tato vlastnost pak umožňuje poměrně snadno dokázat některé známé výsledky z teorie zpožděných diferenciálních rovnic, které byly původně získány jinou cestou. Navíc skrze ni vede cesta k numerickému vyčíslení funkčních hodnot v ryze reálné aritmetice. Nabízí se tedy alternativa ke standardním numerickým algoritmům pracujícím v komplexní aritmetice. Výše nastíněným otázkám se bude příspěvek věnovat.

A note on the D-QSSA method with optimal constant delays applied to a class of mathematical models

Štěpán Papáček, Ctirad Matonoha

We develop and test a modification of an enhancement of the classical model reduction method applied to a system of chemical reactions. The enhanced method called as the delayed quasi-steady-state approximation (D-QSSA) method was proposed by Vejchodský et al. (2014). Here, the modification consisting of setting constant delays in some sense optimal, first presented in Matonoha et al. (2022) is further analyzed on the paradigmatic example of the Michaelis-Menten kinetics with a simple transport process. The behavior of the full non-reduced system is compared with several respective variants of reduced models. Finally, the possibility of speeding up the numerical calculations using the Bohl-Marek quasi-linear formulation is highlighted.

Accurate error estimation in (P)CG and CG-like methods

Gerard Meurant, Jan Papež, and Petr Tichý

In [Meurant, Papež, Tichý; Numerical Algorithms 88, 2021], we presented an adaptive estimate for the energy norm of the error in the conjugate gradient (CG) method. The estimate is very cheap to evaluate and numerically reliable in finite-precision computations. Additionally, we show the extension of the estimate for other methods (CGLS, CGNE, LSQR, and CRAIG) for solving linear approximation problems with a general rectangular matrix. The estimate can be used also for preconditioned variants of the methods and can be easily implemented into existing codes.

The core problem story

Martin Plešinger

Přednáška se bude točit kolem tzv. úplných nejmenších čtverců (TLS), snad spíše statistické metody určené k hledání přibližného řešení lineárních aproximačních úloh, ovšem pod algebraickým drobnohledem. V úvodu se seznámíme s tím co a jak metoda dělá, proč někdy nemá odpovídající minimalizační úloha řešení (na rozdíl od obyčejných nejmenších čtverců) a že algoritmy považované za klasické mohou řešení minout i když shodou okolností existuje. Ústředním nástrojem, který nám pomůže vnést do problematiky řád, bude koncept tzv. core problému — minimálního podproblému, který obsahuje informaci nutnou a postačující k řešení úlohy původní. Řád nicméně nebude trvat věčně a i nepatrné zobecnění původně vektorové úlohy do světa matic vezme core problému vítr z plachet — i zde se objeví problémy s řešitelností. Nás bude zajímat proč tomu tak je. Nebo alespoň odkud a jak se na úlohu můžeme dívat abychom snad původ neexistence řešení spatřili, nebo alespoň odkud jsme se snažili dívat my...

How to express the solution of an ODE as a linear system (in a suitable algebra) and exploit it for fast computation

Stefano Pozza

The solution of systems of non-autonomous linear ordinary differential equations is crucial in various applications, such as nuclear magnetic resonance spectroscopy. We introduced a new solution expression in terms of a generalization of the Volterra composition. Such an expression is linear in a particular algebraic structure of distributions, which can be mapped onto a subalgebra of infinite matrices. It is possible to exploit the new expression to devise fast numerical methods for linear non-autonomous ODEs. As a first example, we present a new method for the operator solution of the generalized Rosen-Zener model, a system of linear non-autonomous ODEs from quantum mechanics. The new method's computing time scales linearly with the model's size in the numerical experiments.

Comparison of FETI methods applied to the problem of linear elasticity

Adam Růžička, David Horák

We present the results from the numerical experiments performed on a simple 2D model problem of linear elasticity solved by variants of the FETI method, specifically FETI-1, TFETI-1, (T)FETI-2, FETI-DP and HTFETI-1. (T)FETI-2 is regarded here as (T)FETI-1 with deflation applied on the CG method solving the obtained final system of linear equations. Various methods of deflation are tested and compared, specifically enforcing the equality of the displacement of the corresponding corner nodes of adjacent subdomains, enforcing the equality of means of the nodes‘ displacements on opposite sides of the adjacent subdomains‘ interface, eigenvectors of the coefficient matrix of the final system of equations, and a fast discrete wavelet transform of this coefficient matrix. In experiments performed using HTFETI-1, two variants of aggregating the subdomains in the clusters are tested and compared – by conditions of equality of displacements of corresponding corner nodes of adjacent subdomains in the same cluster and by conditions of equality of means of displacements on opposite sides of the interface of adjacent subdomains in the same cluster. For both TFETI-1 and HTFETI-1, various ways of connecting the corresponding quartets of corner nodes of 4 adjacent subdomains are analyzed as well.
Both the numbers of CG iterations of the CG method solving the final system of linear equations and the spectral properties of the dual operators corresponding to the respective tested standard, preconditioned, or deflated variants of the FETI method are investigated in this material.

Afinně posunuté lineární kódy a jejich vlastnosti v binárním symetrickém kanálu

Adam Rychtář

K ověření integrity přenesených zpráv v komunikačních systémech v bezpečnostně relevantních aplikacích se používají lineární detekční kódy. Přidání implicitní informace se realizuje závěrečnou modifikací zprávy, což odpovídá systému afinních posunutí lineárního kódu. Příspěvek se zabývá vhodným návrhem systémů posunutí detekčních kódů, které splňují bezpečnostní kvantitativní cíle, zejména pravděpodobnost vzniku nedetekovatelné chyby a záměny přidané implicitní informace v binárním symetrickém kanálu.

Spherical RBF interpolation employing particular geodesic metrics and trend functions

Karel Segeth

The contribution is concerned with the spherical radial basis function approximation of scalar physical data measured on the surface of a unit sphere in 3D. We use several geodesic metrics to obtain spherical radial basis functions and the approximation formulas with only a single trend function that corresponds to the theoretical model, in particular a polynomial of degree 2 considered in Cartesian coordinates. We prove the existence of the interpolation formula of the type considered.
On simple computational examples we present a comparison of the influence of different geodesic metrics on the quality of results.

Synergy of adaptive coarse space and Krylov subspace recycling for the BDDC method

Martin Hanek, Jan Papež, and Jakub Šístek

Means of acceleration of iterative methods for sequences of linear systems have been extensively studied in literature. A widely used approach is recycling the subspace within a Krylov method combined with deflation. Another approach is based on improving the preconditioner. In domain decomposition methods, adaptive selection of coarse space is the state of the art leading to powerful preconditioners. We compare these two approaches and study their combination for unsteady incompressible flow problems governed by the Navier-Stokes equations. These are solved by the pressure-correction scheme in connection with the finite element method. This approach leads to sequences of linear systems over the time steps. Our particular interest is the Poisson problem of pressure. Results for the problem of flow behind the sphere for Reynolds number 300 are presented. We demonstrate that by using these approaches we are able to save about half of the computational time.

Hydro-mechanical model of fractured rocks with contact conditions

Jan Stebel

We present a numerical model of flow and mechanics in porous media that takes into account the presence of fracture networks. Fractures are modelled as lower-dimensional poroelastic manifolds which communicate with the surrounding rock through appropriate interface conditions. In particular, the non-penetration contact conditions are considered. Further nonlinearities such as the cubic law of fracture conductivity are considered. We describe the finite element discretization, the fixed-stress iterative splitting and the implementation into the in-house solver Flow123d with help of the PERMON library for solving contact mechanical problems. The robustness and efficiency of the method is demonstrated on a benchmark problem of tunnel excavation. The results were obtained jointly with J. Březina (TUL), J. Kružík, D. Horák and M. Béreš (Institute of geonics, AS CR).

On mathematical modelling and numerical simulations of fluid-structure interaction problems in biomechanics of voice production

Petr Sváček

In this paper we focus on mathematical modeling of human phonation. The mechanism of voice production is a highly complex multiphysical process. During the phonation the fundamental sound is created by vibrations of vocal folds induced by the airflow coming from the human lungs. This primary sound is further modulated by the geometry of the vocal tract corresponding to a pronounced vowel. The phonation process involves interactions of the air flow, self-sustained oscillations of the vocal folds and acoustics. Vocal folds oscillations excited by the airflow are usually limited by their contacts during voicing, which results in the complete closure of the glottis. In this paper the problem of mathematical modelling and its numerical approximation of this process is discussed.

Optimalizace prvků 2D rámové konstrukce s využitím Inexact restoration

Tadeáš Světlík

Příspěvek se věnuje optimalizaci hmotnosti prvků dvoudimenzionálních prutových konstrukcí s ohledem na nezbytné požadavky na použitelnost a únosnost, jako jsou limitní deformace a napětí. Konstrukce je pro účely výpočtu diskretizována a analyzována pomocí metody konečných prvků, přičemž každý prvek je specifikován svými průřezovými charakteristikami, včetně plochy průřezu, čímž je zároveň definována jeho hmotnost. Optimalizační úloha spočívá v minimalizaci nelineární hmotnostní funkce se silně nelineárním nerovnostním omezením, na kterou je aplikován algoritmus Inexact Restoration. V příspěvku je stručně popsána metoda konečných prvků a její uplatnění na prutové konstrukce, dále je představen optimalizační algoritmus Inexact Restoration a jeho výsledky jsou demonstrovány na příkladu s analytickým řešením.

Advanced algorithms for slope stability analysis (Pokročilé algorithmy pro analýzu stability svahů)

Stanislav Sysala

This contribution is focused on numerical determination of factors of safety (FoS) within slope stability assessment. We define FoS either by the limit load analysis (LA) or the shear strength reduction method (SSRM). The main aim of this contribution is to explain this problematic on an algebraic level to be the topic understandable for broader class of scientists and our algorithms more transparent. To this end we introduce a system of nonlinear equations and specify the assumptions on the corresponding nonlinear operator. Then, two different parametrizations of the system are considered and defined using a scalar factor. Its critical value represents FoS either for LA or SSRM. We present advanced continuation techniques and Newton-like methods enabling to determine FoS. The suggested algorithms are illustrated on slope stability numerical examples. The corresponding codes in Matlab are available for download.

Numerical investigation of the fluid-structure interaction problems using finite element method: comparison between Scott-Vogelius and Taylor-Hood elements

Karel Vacek, Petr Sváček

This paper is interested in numerical approximation of fluid-structure interaction (FSI) problems. The fluid flow is considered as incompressible, and due to the problems including moving domain, the Arbitrary Eulerian-Lagrangian (ALE) method is employed. The fluid is modelled by the Navier-Stokes equations in the ALE formulation, which is developed through a pseudo-elastic technique, and the fluid flow is discretized by the Finite Element Method (FEM). This research mainly focuses on the comparison of the performance of two types of finite elements: the Taylor-Hood (TH) element and the Scott-Vogelius (SV) element. Both elements meet the Babuška-Brezzi (BB) inf-sup condition, ensuring the numerical scheme's stability. The TH element satisfies the BB condition on the regular mesh, whereas, for the SV element, a mesh derived from barycentric refinement of regular triangulation is employed to fulfil the BB stability. The paper presents numerical results from two benchmark tests of flow around a cylinder and flow around the NACA0012 airfoil, to demonstrate the effectiveness of these methods.

K možnostem matematického modelování interakce stavby s podložím

M. Jedlička, I. Němec, J. Vala

Při statické i dynamické analýze napjatosti a přetvoření zemního masivu/podloží do základové konstrukce se nabízejí následující možnosti redukce dimenze: i) na základě efektivního modelu podloží Kolář & Němec (1989), vycházejícího z Pasternakova přístupu (1936, 1954), jenž umožňuje dopočítat dvojici materiálových parametrů povrchového modelu na základě energetické ekvivalence, v dalším kroku je pak nahrazeno podloží mimo základovou spáru vlastnostmi linie na okraji základové spáry; ii) redukcí rozsáhlé pásové matice tuhosti zemního masivu do menší plné matice tuhosti konstrukce s využitím Schurova doplňku. V obou případech je rozhodující kombinace, pro niž je konstrukce ustálená; další zatěžovací stavy, jež nemají zásadní vliv na podloží, lze dopočítávat bez nutnosti provádění celkového výpočtu. Diskutována je vhodnost použití algoritmů i), ii) pro konkrétní typy konstrukcí.

Mathematical modelling of human phonation process

Jan Valášek

In this talk the fluid-structure-acoustic interaction (FSAI) problem with a particular interest in application on the problem of human phonation is addressed. This problem consists of the three physical fields: a highly complex airflow, a vibrating elastic structure (vocal folds) and acoustics and their mutual interactions. A general mathematical model of the FSAI problem will be described and the later simplifications of it will be discussed. Since the main sound source of the human phonation is generated by an unsteady flow in the complex configuration of vocal tract the aeroacoustics as a separate branch of acoustics will be introduced. The ultimate goal of the aeroacoustics is to establish a well solvable simplification of the full description provided by the Navier-Stokes equations (for compressible fluid). Due to recent advantages the computational aeroacoustics has attained also a high interest in the field of technical applications. In the end the numerical simulations of flow-induced vibrations of vocal folds and the propagation of sound through human vocal tract based on the previously mentioned approaches will be shown.

Metoda diskrétních prvků pro problematiku trhlin s využitím Newmark-beta metody

Radek Varga

Příspěvek se zaměřuje na aplikaci a porovnání newmarkových beta metod pro metodu diskrétních prvků rozšířenou o model BBM (beam bound model), který umožňuje modelování kombinovaných kontinuálních a partikulárních problémů. V rámci výzkumu je tato metoda aplikována na problematiku šíření trhlin v betonových a železobetonových materiálech, jakožto alternativa k běžně využívaným metodám FEM.